THE EFFICIENCY OF THE FIN

In what follows, we shall restrict ourselves to the case where there is no heat loss at the tip of the fin; the direction of fluid flow needs not to be indicated anymore.

The classical concept of efficiency introduced by Harper and Brown [1] can now be introduced: this "constitution efficiency" ε_k is the ratio of the heat flux dissipated by the fin to the one which would be transferred if the heat conductivity of the fin had become infinite. If $k \to \infty$, then $Bi \to 0$, $R \to 1$, and we have:

$$\varepsilon_k = \frac{1+\dfrac{1}{\tanh\dfrac{\sigma^*}{2}}}{1+\dfrac{R}{\tanh\dfrac{R\sigma^*}{2}}}.$$

When, furthermore, the heat capacity rate becomes infinite ($\sigma^* \to 0$), so as it is supposed in most cases, we find the classical expression:

$$\lim_{Pc\to\infty} \varepsilon_k = \frac{\tanh H\sqrt{(2Bi)}}{H\sqrt{(2Bi)}}.$$

But we can find other relevant criteria of perfectness, which may characterize the property of efficiency of a fin. Particularly, the parameter ε^* that we have emphasized here above appears to be a "global efficiency" of the fin, since the fin of reference should have both infinite thermal conductivity and height or extension.

Another interesting reference should be a fin of infinite extension but with its real thermal conductivity: the "extension efficiency" defined in this scope is of interest for the designer [2].

With finite fluid flow, the "extension efficiency" ε_h is given by:

$$\varepsilon_h = \frac{1+R}{1+\dfrac{R}{\tanh\dfrac{R\sigma^*}{2}}},$$

while, when fluid flow becomes infinite, the following and very simple expression is found:

$$\lim_{Pc\to\infty} \varepsilon_h = \tanh H\sqrt{(2Bi)}.$$

CONCLUSIONS

We have solved the most general problem of heat transfer from fins with heat losses at their tips and finite fluid flow between them. This was done in the scope of the concept of effectiveness of a heat-transfer cell. From the results first obtained, we have deduced general expressions for several parameters of efficiency of a fin.

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ON THE LINEARIZED ANALYSIS OF ENTRANCE FLOW IN HEATED, POROUS CONDUITS

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NOMENCLATURE

C_n, D_n, constant coefficients;
$E_1(x), E_2(x)$, residual terms;
$F_n(y)$, function defined by (13);
Nu, Nusselt number;
Pe, Peclet number;
Pr, Prandtl number;
Re, Reynolds number;
T, temperature;
U, mean velocity;
u, v, velocity components in x, y directions;
x, y, coordinates parallel to and normal to flow direction;
ξ, reduced coordinate of x, (9);
λ_n, β_n, eigenvalues, (18), (27);
ϕ, confluent hypergeometric function.

Superscripts
b, bulk;
i, inlet of conduit;
w, wall of conduit.

The steady, laminar, incompressible flow in the entrance region of heated, porous conduits is investigated by the linearized method, which is known to yield good results for momentum transfer in tubes of impermeable wall but is only fairly good for momentum transfer and failed for heat transfer in porous tubes [1]. It is shown in this note that the method gives also good results for porous conduits and analytical solutions can be obtained, provided that the transverse velocity is taken into account and suitably approximated.

LINEARIZED EQUATIONS

Consider the laminar flow of an incompressible fluid between two parallel semi-infinite porous plates and through a semi-infinite circular tube of porous wall. As usual, all thermo-physical properties of the fluid are assumed constant, and the rate of injection or suction and the wall temperatures are assumed constant and uniform. The inlet velocity and temperature profiles are prescribed: uniformly distributed over the cross-section or fully-developed in conduits of permeable or impermeable walls, or any other symmetric profile.

The conservation equations of mass, momentum and energy are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{mv}{y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{2}\frac{\mathrm{d}p}{\mathrm{d}x}+\frac{1}{Re}\frac{1}{y^m}\frac{\partial}{\partial y}\left(y^m\frac{\partial u}{\partial y}\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{Pe}\frac{1}{y^m}\frac{\partial}{\partial y}\left(y^m\frac{\partial T}{\partial y}\right) \tag{3}$$

where $m = 0, 1$ for the two-dimensional and the axially symmetric flow, respectively; u and v are the velocity components in x and y directions; p is the pressure; T the temperature excess over the wall temperature; Re and Pe are Reynolds and Peclet numbers defined on the basis of inlet mean-velocity and one half of the spacing for parallel plates or the radius of circular tubes. All quantities in equations (1)–(3) have been normalized; velocity by the inlet mean-velocity; pressure by the inlet dynamic-pressure; temperature by the difference between the inlet mean and wall temperatures; and x and y by y_0 is the inside radius of the tube or the distance between the mid-plane ($y = 0$) and the channel wall. Equations (1)–(3) are of the boundary layer type of which the domain of validity has been discussed elsewhere such as [1, 2].

The appropriate boundary conditions on $u(x, y)$ and $T(x, y)$ are:

$$u(x,1)=0,\quad v(x,1)=-v_w,\quad \frac{\partial}{\partial y}u(x,0)=0,\quad u(0,y)=u_i(y) \tag{4}$$

$$T(x,1)=0,\quad \frac{\partial}{\partial y}T(x,0)=0,\quad T(0,y)=T_i(y) \tag{5}$$

where v_w is the velocity at the wall, positive for injection and negative for suction, and $u_i(y)$ and $T_i(y)$ are the inlet velocity and temperature all having been normalized by appropriate quantities as indicated earlier.

We now propose the linear approximation of the convective terms in equations (2) and (3) as follows:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U(x)\frac{\partial u}{\partial x}+V(y)\frac{\partial u}{\partial y}+E_1(x) \tag{6}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=U(x)\frac{\partial T}{\partial x}+V(y)\frac{\partial T}{\partial y}+E_2(x) \tag{7}$$

where $E_1(x)$ and $E_2(x)$ are the residues; $U(x)$ is the local mean velocity; and $V(y)$ is assumed to be a linear function of the transverse coordinate satisfying the conditions $V(0) = 0$ and $V(1) = -v_w$:

$$V(y)=-v_w y. \tag{8}$$

Introducing a new coordinate

$$\xi=\frac{1}{Re}\int_0^1\frac{\mathrm{d}x}{1+v_w x}=\frac{1}{Re_w}\ln U(x) \tag{9}$$

we obtain the linearized equations of momentum and energy:

$$\frac{\partial u}{\partial \xi}-Re_w y\frac{\partial u}{\partial y}-\frac{1}{y^m}\frac{\partial}{\partial y}\left(y^m\frac{\partial u}{\partial y}\right)+2^m\left(\frac{\partial u}{\partial y}\right)_{y=1}=2^{m+1}Re_w\,\mathrm{e}^{2^m Re_w\xi} \tag{10}$$

$$\frac{\partial T}{\partial \xi}-Re_w y\frac{\partial T}{\partial y}-\frac{1}{Pry^m}\frac{\partial}{\partial y}\left(y^m\frac{\partial T}{\partial y}\right)=0. \tag{11}$$

The boundary conditions on u and T remain in the same forms as (4) and (5) with x replaced by ξ.

SOLUTION FOR VELOCITY

We seek the solution for $u(\xi, y)$ in the form

$$C_n Y_n(y)\mathrm{e}^{-\lambda_n\xi}$$

where C_n and λ_n are arbitrary constants and $Y_n(y)$ satisfies

$$Y_n''(y)+\left(Re_w y+\frac{m}{y}\right)Y'(y)+\lambda_n Y_n(y)-2^m Y_n'(1)=0 \tag{12}$$

$$Y_n(1)=Y_n'(0)=0.$$

It can be easily shown that $Y_n(y)$ are orthogonal to the function $F_n(y)$ defined by

$$F_n(y)=Y_n(y)-\frac{2^m}{\lambda_n}Y'(1) \tag{13}$$

with respect to the weight function $w = y^m\exp(Re_w y^2/2)$. Thus, the solution of (10) satisfying the boundary conditions on u in (4) can be written in the form

$$u(\xi,y)=\int_0^1 u_i(y')G(\xi,y\,|\,0,y')y'^m\,\mathrm{d}y'+\int_0^1 G(\xi,y\,|\,\xi',y')Q(\xi')y'^m\,\mathrm{d}y' \tag{14}$$

where $G(\xi, y, \xi', y')$ is given by

$$G(\xi,y\,|\,\xi',y')=\sum_{n=1}^{\infty}\frac{Y(y)F_n(y')\exp(y'^2Re_w/2)}{\int_0^1 Y_n(y')F_n(y')\exp(y'^2Re_w/2)y'^m\,\mathrm{d}y'}\mathrm{e}^{-\lambda_n(\xi-\xi')} \tag{15}$$

and

$$Q(\xi')=2^{m+1}Re_w\,\mathrm{e}^{2^m Re_w\xi'}. \tag{16}$$

If we rewrite (12) in terms of $F''(y)$ and introduce a new variable $z = -Re_w y^2/2$, we can obtain

$$F_n=\phi\left(\frac{\lambda_n}{2Re_w};\frac{m+1}{2};z\right) \tag{17}$$

where ϕ is the confluent hypergeometric function and λ_n are the roots of the transcendental equation

$$\phi\left(\frac{\lambda_n}{2Re_w};\frac{m+1}{2};-\frac{Re_w}{2}\right)-\phi\left(\frac{\lambda_n}{2Re_w}+1;\frac{m+3}{2};-\frac{Re_w}{2}\right)=0. \tag{18}$$

Combining (13) and (17) gives

$$Y_n(y)=\phi\left(\frac{\lambda_n}{2Re_w};\frac{m+1}{2};-\frac{Re_w}{2}y^2\right)-\phi\left(\frac{\lambda_n}{2Re_w};\frac{m+1}{2};-\frac{Re_w}{2}\right). \tag{19}$$

This completed the solution for u in (14) which gives

$$u(\xi,y)=\sum_{n=1}^{\infty}C_n\left[\phi\left(\frac{\lambda_n}{2Re_w};\frac{m+1}{2};-\frac{Re_w}{2}y^2\right)-\phi\left(\frac{\lambda_n}{2Re_w};\frac{m+1}{2};-\frac{Re_w}{2}\right)\right]\times\left\{1+\frac{2Re_w}{Re_w+\lambda_n/(m+1)}\left[\mathrm{e}^{(2^m Re_w+\lambda_n)\xi}-1\right]\right\}\mathrm{e}^{-\lambda_n\xi} \tag{20}$$

where

$$C_n=\frac{2^m\phi\left(\frac{\lambda_n}{2Re_w};\frac{m+1}{2};-\frac{Re_w}{2}\right)\mathrm{e}^{Re_w/2}}{\lambda_n\int_0^1\phi^2\left(\frac{\lambda_n}{2Re_w}+1;\frac{m+3}{2};-\frac{Re_w}{2}y^2\right)\mathrm{e}^{Re_w y^2/2}y^{m+2}\,\mathrm{d}y} \tag{21}$$

for uniform inlet velocity, and

$$\frac{\frac{13m+3}{2}\int_0^1 \phi\left(\frac{\lambda_n}{2Re_w};\ \frac{m+1}{2};\ -\frac{Re_w}{2}y^2\right)e^{Re_w y^2/2}(1-y^2)y^m\,\mathrm{d}y}{\lambda_n\int_0^1 \phi^2\left(\frac{\lambda_n}{2Re_w}+1;\ \frac{m+3}{2};-\frac{Re_w}{2}y^2\right)e^{Re_w y^2/2}\,y^{m+2}\,\mathrm{d}y} \tag{22}$$

for parabolic inlet velocity.

SOLUTION FOR TEMPERATURE

Again we seek the solution for T in the form of

$$D_n\theta_n(y)\,e^{-\beta_n\xi/Pr}$$

where D_n and β_n are arbitrary constants, Pr is the Prandtl number, and θ_n satisfies

$$\theta_n''(y)+\left(Pe_w y+\frac{m}{y}\right)\theta_n'(y)+\beta\theta_n(y)=0 \tag{23}$$

$$\theta_n(1)=0 \tag{24}$$

$$\theta_n'(0)=0. \tag{25}$$

As before, introducing a new independent variable $z=-Pe_w y^2/2$, we obtain the solution for $\theta(y)$:

$$\theta_n(y)=\phi\left(\frac{\beta_n}{2Pe_w};\ \frac{m+1}{2};\ -\frac{Pe_w}{2}y^2\right) \tag{26}$$

where β_n is the root of the transcendental equation

$$\phi\left(\frac{\beta_n}{2Pe_w};\ \frac{m+1}{2};\ -\frac{Pe_w}{2}\right)=0. \tag{27}$$

It can be easily shown that functions $\theta_n(y)$ are mutually orthogonal with respect to the weight function $w(y)=y^m\exp(Pe_w y^2/2)$. Thus, the solution for the temperature is

$$T(\xi,y)=\int_0^1 T_i(y')\boldsymbol{g}(\xi,y\,|\,0,y')y'^m\,\mathrm{d}y' \tag{28}$$

where $\boldsymbol{g}(\xi,y\,|\,0,y')$ is given by

$$\boldsymbol{g}(\xi,y\,|\,0,y')=\sum_{n=1}^{\infty}\frac{\theta_n(y)\theta(y')\,e^{Pe_w y'^2/2}}{\int_0^1\theta_n^2(y)\,e^{Pe_w y^2/2}\,y^m\,\mathrm{d}y}\cdot e^{-\beta_n\xi/Pr}. \tag{29}$$

For the case of uniform inlet temperature, (28) becomes

$$T(\xi,y)=\sum_{n=1}^{\infty}D_n\phi\left(\frac{\beta_n}{2Pe_w};\ \frac{m+1}{2};\ -\frac{Pe_w}{2}y^2\right)e^{-\beta_n\xi/Pr} \tag{30}$$

where

$$D_n=\frac{\phi\left(\frac{\beta_n}{2Pe_w}+1;\ \frac{m+3}{2};\ -\frac{Pe_w}{2}\right)e^{Pe_w/2}}{(m+1)\int_0^1\phi^2\left(\frac{\beta_n}{2Pe_w};\ \frac{m+1}{2};\ -\frac{Pe_w}{2}y^2\right)e^{Pe_w y^2/2}\,y^m\,\mathrm{d}y}. \tag{31}$$

FRICTION AND HEAT-TRANSFER COEFFICIENTS

We normalize the shear stress by the local dynamic pressure to obtain the friction coefficient (C_f) and the pressure drop

$$C_f Re(\xi)=-2\left[\frac{\partial}{\partial y}\left(\frac{u}{U}\right)\right]_{y=1}$$

$$=\frac{2^{1-m}}{U}\sum_{n=1}^{\infty}C_n\lambda_n\phi\left(\frac{\lambda_n}{2Re_w}+1;\ \frac{m+3}{2};\ -\frac{Re_w}{2}\right)e^{-\lambda_n\xi}$$

$$\times\left\{1+\frac{2Re_w}{Re_w+\lambda_n/(m+1)}\left[e^{(2^m Re_w+\lambda_n)\xi}-1\right]\right\} \tag{32}$$

$$p(0)-p(x)=2^{m+1}\left[\int_0^1 u^2\,\mathrm{d}y-A_m+\sum_{n=1}^{\infty}C_n\lambda_n\frac{U^2-U^{(1-\lambda_n/2^m Re_w)}}{(3m+1)Re_w+(m+1)\lambda_n}\times\phi\left(\frac{\lambda_n}{2Re_w}+1;\ \frac{m+3}{2};\ \frac{Re_w}{2}\right)\right] \tag{33}$$

where C_n is given by (21) and (22) and $A_m=(2-m)/2$, $(18-8m)/15$ for uniform and parabolic inlet velocities respectively. We define the local Nusselt number on the basis of $2y_0$,

$$Nu(\xi)=-\frac{2}{T_b}\left(\frac{\partial T}{\partial y}\right)_{y=1}=\frac{2^{1-m}}{T_b}\sum_{n=1}^{\infty}$$

$$\times D_n\beta_n\phi\left(\frac{\beta_n}{2Pe_w}+1;\ \frac{m+3}{2};\ -\frac{Pe_w}{2}\right)e^{-\beta_n\xi/Pr} \tag{34}$$

where D_n is given by (31), and T_b is the normalized bulk temperature.

CALCULATED RESULTS

Calculated results of velocity and temperature distributions, friction and heat-transfer coefficients and pressure drops are found in good agreement with those obtained from numerical solution of the conservative equations for flow in two parallel plates [3] and in a tube [4]. However, only some of the velocity and temperature distributions are reported here as shown in Figs. 1 and 2 where ξ has been converted to $x^+=x/Re$. For x^+ larger than a certain value, the velocity and temperature profiles and Nusselt number remain essentially unchanged. On the basis of 1 per cent deviation, the fully developed Nusselt numbers are shown in Table 1.

Table 1. Fully developed Nusselt numbers for parallel plates (I) and circular tubes (II) with $Pr=0{\cdot}72$ and uniform inlet velocity

Re_w	Present (I)	Doughty (I)	Terrill–Walker (I)	Present (II)	Kinney (II)
−5	16·51	16·93	17·12		
−1	9·50	9·09	9·09	4·78	4·45
1	6·68	6·16	6·17	3·63	3·20
5	3·28	2·40	2·52	1·96	1·30
10	1·13	0·54	2·06	0·77	0·30

Results reported in [3, 5, 6] are also reproduced in Table 1 for comparison.

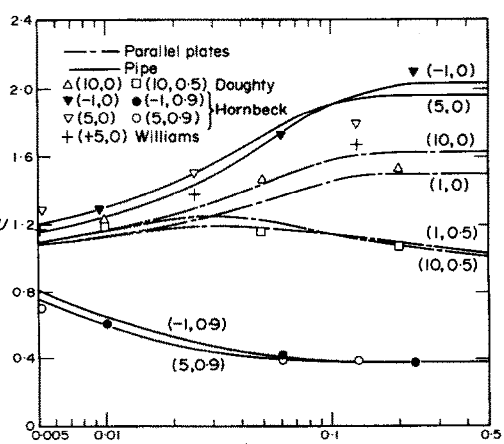

FIG. 1. Axial velocity variation (Re_w, y) with uniform inlet velocity.

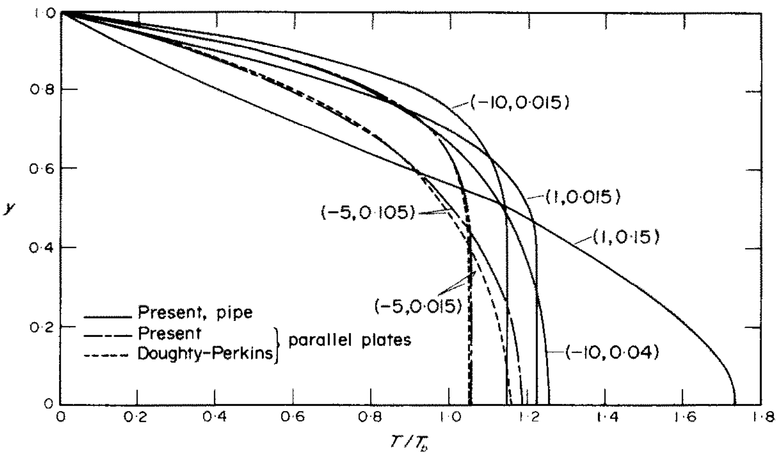

FIG. 2. Temperature distribution (Re_w, x^+) with uniform inlet velocity, $Pr = 0{\cdot}72$.

The numerical calculation of confluent hypergeometric functions is simple and can be tabulated once and for all and the series solutions for the velocity and temperature converge quite rapidly. The method is particularly convenient for different initial velocities and temperatures and for fluids of different Prandtl numbers. For Prandtl number other than 0·72, the temperature profile is the same for the same wall Peclet number provided that $x^+(0{\cdot}72)$ is replaced by $x^+(Pr)$ defined by

$$x^+(Pr) = \frac{0{\cdot}72}{Pe_w}\{[1+Re_w x^+(0{\cdot}72)]^{Pr/0{\cdot}72}-1\}.$$

It can be concluded that the linearized method which has been employed with remarkable success for flow in ducts of impermeable walls is also useful for conduits of porous walls, heated or unheated, provided that the transverse convective terms in the momentum and energy equations are taken into account. Evidently, the method can be applied to problems in other physical contexts such as those cited in [1]. It should also be remarked that more accuracy can be obtained if we make another correction on $U(x)$ as was done in [7], either through the momentum or energy equation. However, the improvement is gained at the expense of the simplicity of the method.

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HEAT TRANSFER AT THE CONDENSATION OF STEAM ON TURBULENT WATERJET

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NOMENCLATURE

α, heat-transfer coefficient;
v, velocity of water jet;
A, the total cross-section of water jet at the mean water velocity;
F, the total surface of water jet;
S, the total cross-section of water jet at the outpouring;
t_s, saturation temperature;
t_{win}, temperature of the entering water;
t_{wout}, temperature of the outgoing water;
ρ, density of water;
c, heat capacity of water;
r, heat of evaporation;
K, $= r/c{\cdot}(t_s - t_{win})$;
We, Weber number.